

Introduction - Answers

General Probability Questions

1) If there are N students taking COMP 3331/9331, what is the probability that none of them has a birthday on the first day of the class? (Assume there are exactly 365 days in a year)

Answer: Probability that one student does not have a birthday on the first day is $(364/365)$. Since there are N students, the required probability will be $(364/365)^N$.

2) How many students would there need to be in the class for the probability in (1) to be less than 50%?

Answer: $(364/365)^N < 0.50$

$$N \times \log_{10} (364/365) < \log_{10} (0.5)$$

$$N > 252.65 \Rightarrow N \geq 253 \text{ students}$$

3) What is the probability that five years running, no student has a birthday on the first day of the class? (Assume the same number of students take the class each year, and no one repeats the class)

Answer: The five years should be viewed as independent events, and so we multiply the probability for all the five years, resulting in the following solution: $(364/365)^{5N}$

4) Would your answer to (3) above be larger or smaller if one or more students re-took the class within the five-year period?

Answer: The probability would be *smaller*. Consider the following example. Assume that there is always one student who retakes the class starting from the second year. In other words, the number of students for year 2, 3, 4 and 5 are $N+1$. So in this case, the probability for these subsequent years will be $(364/365)^{N+1}$. The required probability will thus be: $(364/365)^N * (364/365)^{4(N+1)}$ which is smaller than the answer for question 3.

Networking Questions

Q1) What is meant by the term statistical multiplexing?

Answer: In statistical multiplexing, data from multiple users (senders) is sent over a link. If one user does not use its share of the bandwidth, it is then free to be used by other users. Thus, senders share the link bandwidth, with no user having all of the link bandwidth allocated to it.

Q2) Consider two hosts, A and B, connected by a single link of rate R bps. Suppose that the two

hosts are separated by m meters, and suppose the propagation speed along the link is s meters/sec. Host A is to send a packet of size L bits to Host B.

- (a) Express the propagation delay, d_{prop} in terms of m and s .

Answer: $d_{prop} = m/s$

- (b) Determine the transmission time of the packet, d_{trans} in terms of L and R .

Answer: $d_{trans} = L/R$

- (c) Ignoring the processing and queuing delays, obtain an expression for the end-to-end delay.

Answer: End-to-end delay is $d_{prop} + d_{trans} = m/s + L/R$

- (d) Suppose Host A begins to transmit the packet at time $t=0$. At time $t=d_{trans}$, where is the last bit of the packet?

Answer: The bit is just leaving host A.

- (e) Suppose d_{prop} is greater than d_{trans} . At time $t= d_{trans}$, where is the first bit of the packet?

Answer: The first bit is on the link and has not yet reached host B.

- (f) Suppose d_{prop} is less than d_{trans} . At time $t= d_{trans}$, where is the first bit of the packet?

Answer: The first bit has reached host B.

Q3) Recall that in Q2 propagation delay is expressed by m/s . Thus, it does not depend on the bandwidth. Answer is false.

Q4) Suppose users share a 1Mbps link. Also suppose each user requires 100 kbps when transmitting, but each user transmits only 10 percent of the time.

- (a) When circuit switching is used, how many users can be supported?

Answer: 10 users can be supported with circuit switching.

- (b) Suppose packet switching is used for the rest of the problem. Find the probability that a given user is transmitting.

Answer: $p=0.1$

- (c) Suppose there are 40 users. Find the probability that at any given time, exactly n users are transmitting simultaneously.

Answer:

$$\binom{40}{n} p^n (1-p)^{40-n}$$

Q5) Suppose there is exactly one packet switch between a sending host and the receiving host. Assume that the transmission speed of the links between the sending host and the switch and the switch and the receiving host are R_1 and R_2 respectively. Assuming that the switch uses store-and-forward packet switching, what is the total end-to-end delay to send a packet of length L ? Ignore, queuing, propagation and processing delays.

Answer: At time t_0 the sending host begins to transmit. At time $t_1 = L/R_1$, the sending host completes transmission and the entire packet is received at the router (no propagation delay). Because the router has the entire packet at time t_1 , it can begin to transmit the packet to the receiving host at time t_1 . At time $t_2 = t_1 + L/R_2$, the router completes transmission and the entire packet is received at the receiving host (again, no propagation delay). Thus, the end-to-end delay is $L/R_1 + L/R_2$.

Q6) Review the car-caravan analogy in Section 1.4 of the textbook. Assume a propagation speed of 100 km/hr.

(a) Suppose the caravan travels 200km, beginning in front of one tollbooth, passing through a second tollbooth and finishing just before a third tollbooth. What is the end-to-end delay?

Answer: Tollbooths are 100 km apart, and the cars propagate at 100km/hr. A tollbooth services a car at a rate of one car every 12 seconds. There are ten cars. It takes 120 seconds, or two minutes, for the first tollbooth to service the 10 cars. Each of these cars has a propagation delay of 60 minutes before arriving at the second tollbooth. Thus, all the cars are lined up before the second tollbooth after 62 minutes. The whole process repeats itself for travelling between the second and third tollbooths. Thus the total delay is 124 minutes.

(b) Repeat (a), now assuming that there are seven cars in the caravan instead of 10.

Answer: Delay between tollbooths is 7×12 seconds plus 60 minutes, i.e., 61 minutes and 24 seconds. The total delay is twice this amount, i.e., 122 minutes and 48 seconds.

Q7) Consider sending a large file of F bits from Host A to Host B. There are two links (and one router) between A and B, and the links are uncongested (that is, no queuing delays). Host A segments the file into segments of S bits each and adds 40 bits of header to each segment, forming packets of $L = 40 + S$ bits. Each link has a transmission rate of R bps. Find the value of S that minimizes the delay of moving the file from Host to Host B. Disregard propagation delay.

Answer:

Time at which the 1st packet is received at the destination = $\frac{S + 40}{R} \times 2$ sec. After this, one packet is received by the destination every $\frac{S + 40}{R}$ sec because packets are transmitted back to back by Host A.

Thus delay in sending the whole file is,

$$\text{delay} = \frac{S + 40}{R} \times 2 + \left(\frac{F}{S} - 1\right) \times \left(\frac{S + 40}{R}\right) = \frac{S + 40}{R} \times \left(\frac{F}{S} + 1\right)$$

To calculate the value of S which leads to the minimum delay, we take the derivative and equate it to zero,

$$\frac{\partial \text{delay}}{\partial S} = 0 \Rightarrow \frac{F}{R} \left(\frac{1}{S} - \frac{40 + S}{S^2}\right) + \frac{1}{R} = 0 \Rightarrow S = \sqrt{40F}$$

Q8) In this problem we consider sending real-time voice from Host A to Host B over a packet-switched network (VoIP). Host A converts analog voice to a digital 64kbps bit stream on the fly. Host A then groups the bits into 48-byte packets. There is one link between Host A and B; its transmission rate is 1 Mbps and its propagation delay is 2msec. As soon as Host A gathers a packet, it sends it to Host B. As soon as Host B receives an entire packet, it converts the packet’s bits to an analog signal. How much time elapses from the time a bit is created (from the original analog signal at Host A) until the bit is decoded (as part of the analog signal at Host B)?

Answer: Consider the first bit in a packet. Before this bit can be transmitted, all of the bits in the packet must be generated. This requires

$$\frac{48 \cdot 8}{64 \times 10^3} \text{ sec} = 6 \text{ msec.}$$

The time required to transmit the packet is

$$\frac{48 \cdot 8}{1 \times 10^6} \text{ sec} = 384 \mu \text{ sec.}$$

Propagation delay = 2 msec.

The delay until decoding is

$$6 \text{ msec} + 384 \mu \text{ sec} + 2 \text{ msec} = 8.384 \text{ msec}$$

A similar analysis shows that all bits experience a delay of 8.384 msec.

Q9) Suppose Alice and Bob are sending packets to each other over a computer network. Suppose Trudy positions herself in the network so that she can capture all packets sent by Alice and send whatever she wants to Bob; she can also capture all packets sent by Bob and send whatever she wants to Alice. List some of the malicious things Trudy can do from this position.

Answer: Trudy can pretend to be Bob to Alice (and vice-versa) and partially or completely modify the message(s) being sent from Bob to Alice. For example, she can easily change the phrase “Alice, I owe you \$1000” to “Alice, I owe you \$10,000”. Furthermore, Trudy can even drop the packets that

are being sent by Bob to Alice (and vice-versa), even if the packets from Bob to Alice are encrypted.

Q10) Consider the queuing delay in a router buffer (preceding an outbound link). Suppose all packets are L bits, the transmission rate is R bps and that N packets simultaneously arrive at the buffer every LN/R seconds. Find the average queuing delay of a packet. You can assume that the buffer is empty before the arrival of the first batch of N packet.

Answer: It takes LN/R seconds to transmit N packets. Thus, the buffer is empty when a fresh batch of N packets arrive.

The first of the N packets has no queuing delay. The 2nd packet has a queuing delay of L/R . In general, the n^{th} packet has a delay of $(n-1)L/R$ seconds.

The average delay is

$$\frac{1}{N} \sum_{n=1}^N (n-1)L/R = \frac{L}{R} \frac{1}{N} \sum_{n=0}^{N-1} n = \frac{L}{R} \frac{1}{N} \frac{(N-1)N}{2} = \frac{L}{R} \frac{(N-1)}{2}.$$